SINGLE PHASE FULL WAVE AC VOLTAGE CONTROLLER (AC REGULATOR)

Single phase full wave ac voltage controller circuit using two SCRs or a single triac is generally used in most of the ac control applications. The ac power flow to the load can be controlled in both the half cycles by varying the trigger angle $'\alpha'$.

The RMS value of load voltage can be varied by varying the trigger angle $'\alpha'$. The input supply current is alternating in the case of a full wave ac voltage controller and due to the symmetrical nature of the input supply current waveform there is no dc component of input supply current i.e., the average value of the input supply current is zero.

A single phase full wave ac voltage controller with a resistive load is shown in the figure below. It is possible to control the ac power flow to the load in both the half cycles by adjusting the trigger angle $'\alpha'$. Hence the full wave ac voltage controller is also referred to as to a bi-directional controller.

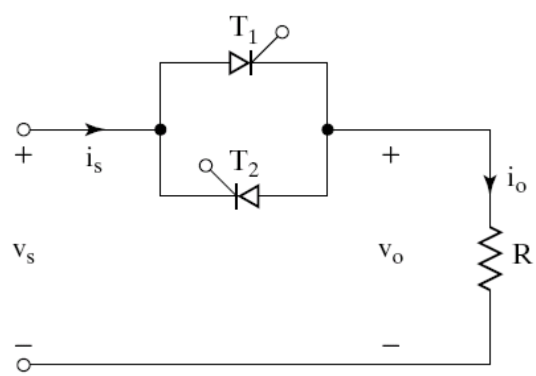


Fig.: Single phase full wave ac voltage controller (Bi-directional Controller) using SCRs

The thyristor T_1 is forward biased during the positive half cycle of the input supply voltage. The thyristor T_1 is triggered at a delay angle of $'\alpha'$ $(0 \le \alpha \le \pi$ radians$)$. Considering the ON thyristor T_1 as an ideal closed switch the input supply voltage appears across the load resistor R_L and the output voltage $v_O = v_S$ during $\omega t = \alpha$ to π radians. The load current flows through the ON thyristor T_1 and through the load resistor R_L in the downward direction during the conduction time of T_1 from $\omega t = \alpha$ to π radians.

At $\omega t = \pi$, when the input voltage falls to zero the thyristor current (which is flowing through the load resistor R_L) falls to zero and hence T_1 naturally turns off . No current flows in the circuit during $\omega t = \pi$ to $(\pi + \alpha)$.

The thyristor T_2 is forward biased during the negative cycle of input supply and when thyristor T_2 is triggered at a delay angle $(\pi+\alpha)$, the output voltage follows the negative halfcycle of input from $\omega t=(\pi+\alpha)$ to 2π. When T_2 is ON, the load current flows in the reverse direction (upward direction) through T_2 during $\omega t=(\pi+\alpha)$ to 2π radians. The time interval (spacing) between the gate trigger pulses of T_1 and T_2 is kept at π radians or 180^0. At $\omega t=2\pi$ the input supply voltage falls to zero and hence the load current also falls to zero and thyristor T_2 turn off naturally.

Instead of using two SCR's in parallel, a Triac can be used for full wave ac voltage control.

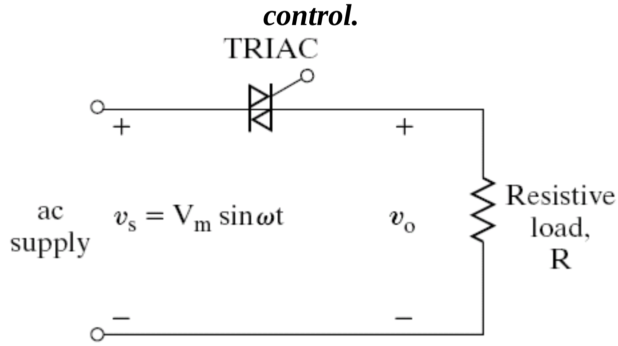


Fig.: Single phase full wave ac voltage controller (Bi-directional Controller) using TRIAC

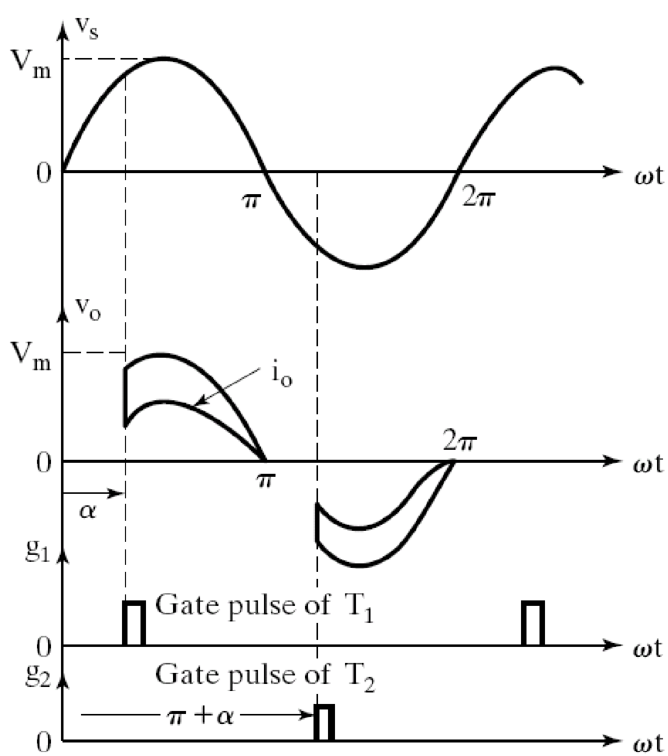


Fig: Waveforms of single phase full wave ac voltage controller

EQUATIONS

Input supply voltage

$$v_S = V_m \sin\omega t = \sqrt{2} V_S \sin\omega t \;;$$

Output voltage across the load resistor R_L;

$$v_O = v_L = V_m \sin\omega t \;;$$

$$\text{for } \omega t = \alpha \text{ to } \pi \text{ and } \omega t = (\pi + \alpha) \text{ to } 2\pi$$

Output load current

$$i_O = \frac{v_O}{R_L} = \frac{V_m \sin\omega t}{R_L} = I_m \sin\omega t \;;$$

$$\text{for } \omega t = \alpha \text{ to } \pi \text{ and } \omega t = (\pi + \alpha) \text{ to } 2\pi$$

TO DERIVE AN EXPRESSION FOR THE RMS VALUE OF OUTPUT (LOAD) VOLTAGE

The RMS value of output voltage (load voltage) can be found using the expression

$$V_{O(RMS)}^2 = V^2_{L(RMS)} = \frac{1}{2\pi}\int_0^{2\pi} v_L^2 d(\omega t)\;;$$

For a full wave ac voltage controller, we can see that the two half cycles of output voltage waveforms are symmetrical and the output pulse time period (or output pulse repetition time) is π radians. Hence we can also calculate the RMS output voltage by using the expression given below.

$$V^2_{L(RMS)} = \frac{1}{\pi}\int_0^{\pi} V_m^2 \sin^2\omega t . d\omega t$$

$$V^2_{L(RMS)} = \frac{1}{2\pi}\int_0^{2\pi} v_L^2 . d(\omega t)\;;$$

$$v_L = v_O = V_m \sin\omega t; \text{ For } \omega t = \alpha \text{ to } \pi \text{ and } \omega t = (\pi + \alpha) \text{ to } 2\pi$$

Hence,

$$V_{L(RMS)}^2 = \frac{1}{2\pi}\left[\int_\alpha^{\pi} (V_m \sin\omega t)^2 d(\omega t) + \int_{\pi+\alpha}^{2\pi} (V_m \sin\omega t)^2 d(\omega t)\right]$$

$$= \frac{1}{2\pi}\left[V_m^2 \int_\alpha^{\pi} \sin^2\omega t . d(\omega t) + V_m^2 \int_{\pi+\alpha}^{2\pi} \sin^2\omega t . d(\omega t)\right]$$

$$= \frac{V_m^2}{2\pi}\left[\int_\alpha^{\pi} \frac{1-\cos 2\omega t}{2} d(\omega t) + \int_{\pi+\alpha}^{2\pi} \frac{1-\cos 2\omega t}{2} d(\omega t)\right]$$

$$=\frac{V_m^{\ 2}}{2\pi\times 2}\left[\int_{\alpha}^{\pi} d\left(\omega t\right)-\int_{\alpha}^{\pi}\cos 2\omega t.d\left(\omega t\right)+\int_{\pi+\alpha}^{2\pi} d\left(\omega t\right)-\int_{\pi+\alpha}^{2\pi}\cos 2\omega t.d\left(\omega t\right)\right]$$

$$=\frac{V_m^{\ 2}}{4\pi}\left[\left(\omega t\right)\Big/_{\alpha}^{\pi}+\left(\omega t\right)\Big/_{\pi+\alpha}^{2\pi}-\left[\frac{\sin 2\omega t}{2}\right]_{\alpha}^{\pi}-\left[\frac{\sin 2\omega t}{2}\right]_{\pi+\alpha}^{2\pi}\right]$$

$$=\frac{V_m^{\ 2}}{4\pi}\left[\left(\pi-\alpha\right)+\left(\pi-\alpha\right)-\frac{1}{2}\left(\sin 2\pi-\sin 2\alpha\right)-\frac{1}{2}\left(\sin 4\pi-\sin 2\left(\pi+\alpha\right)\right)\right]$$

$$=\frac{V_m^{\ 2}}{4\pi}\left[2\left(\pi-\alpha\right)-\frac{1}{2}\left(0-\sin 2\alpha\right)-\frac{1}{2}\left(0-\sin 2\left(\pi+\alpha\right)\right)\right]$$

$$=\frac{V_m^{\ 2}}{4\pi}\left[2\left(\pi-\alpha\right)+\frac{\sin 2\alpha}{2}+\frac{\sin 2\left(\pi+\alpha\right)}{2}\right]$$

$$=\frac{V_m^{\ 2}}{4\pi}\left[2\left(\pi-\alpha\right)+\frac{\sin 2\alpha}{2}+\frac{\sin\left(2\pi+2\alpha\right)}{2}\right]$$

$$=\frac{V_m^{\ 2}}{4\pi}\left[2\left(\pi-\alpha\right)+\frac{\sin 2\alpha}{2}+\frac{1}{2}\left(\sin 2\pi.\cos 2\alpha+\cos 2\pi.\sin 2\alpha\right)\right]$$

$$\sin 2\pi=0 \quad \& \quad \cos 2\pi=1$$

Therefore,

$$V_{L(RMS)}^{2}=\frac{V_m^{\ 2}}{4\pi}\left[2\left(\pi-\alpha\right)+\frac{\sin 2\alpha}{2}+\frac{\sin 2\alpha}{2}\right]$$

$$=\frac{V_m^{\ 2}}{4\pi}\left[2\left(\pi-\alpha\right)+\sin 2\alpha\right]$$

$$V^{2}_{\ L(RMS)}=\frac{V_m^{\ 2}}{4\pi}\left[\left(2\pi-2\alpha\right)+\sin 2\alpha\right]$$

Taking the square root, we get

$$V_{L(RMS)}=\frac{V_m}{2\sqrt{\pi}}\sqrt{\left[\left(2\pi-2\alpha\right)+\sin 2\alpha\right]}$$

$$V_{L(RMS)}=\frac{V_m}{\sqrt{2}\sqrt{2\pi}}\sqrt{\left[\left(2\pi-2\alpha\right)+\sin 2\alpha\right]}$$

$$V_{L(RMS)} = \frac{V_m}{\sqrt{2}}\sqrt{\frac{1}{2\pi}\left[(2\pi - 2\alpha) + \sin 2\alpha\right]}$$

$$V_{L(RMS)} = \frac{V_m}{\sqrt{2}}\sqrt{\frac{1}{2\pi}\left[2\left\{(\pi - \alpha) + \frac{\sin 2\alpha}{2}\right\}\right]}$$

$$V_{L(RMS)} = \frac{V_m}{\sqrt{2}}\sqrt{\frac{1}{\pi}\left[(\pi - \alpha) + \frac{\sin 2\alpha}{2}\right]}$$

$$V_{L(RMS)} = V_{i(RMS)}\sqrt{\frac{1}{\pi}\left[(\pi - \alpha) + \frac{\sin 2\alpha}{2}\right]}$$

$$V_{L(RMS)} = V_S\sqrt{\frac{1}{\pi}\left[(\pi - \alpha) + \frac{\sin 2\alpha}{2}\right]}$$

Maximum RMS voltage will be applied to the load when $\alpha = 0$, in that case the full sine wave appears across the load. RMS load voltage will be the same as the RMS supply voltage $= \frac{V_m}{\sqrt{2}}$. When α is increased the RMS load voltage decreases.

$$V_{L(RMS)}\Big|_{\alpha=0} = \frac{V_m}{\sqrt{2}}\sqrt{\frac{1}{\pi}\left[(\pi - 0) + \frac{\sin 2\times 0}{2}\right]}$$

$$V_{L(RMS)}\Big|_{\alpha=0} = \frac{V_m}{\sqrt{2}}\sqrt{\frac{1}{\pi}\left[(\pi) + \frac{0}{2}\right]}$$

$$V_{L(RMS)}\Big|_{\alpha=0} = \frac{V_m}{\sqrt{2}} = V_{i(RMS)} = V_S$$

The output control characteristic for a single phase full wave ac voltage controller with resistive load can be obtained by plotting the equation for $V_{O(RMS)}$

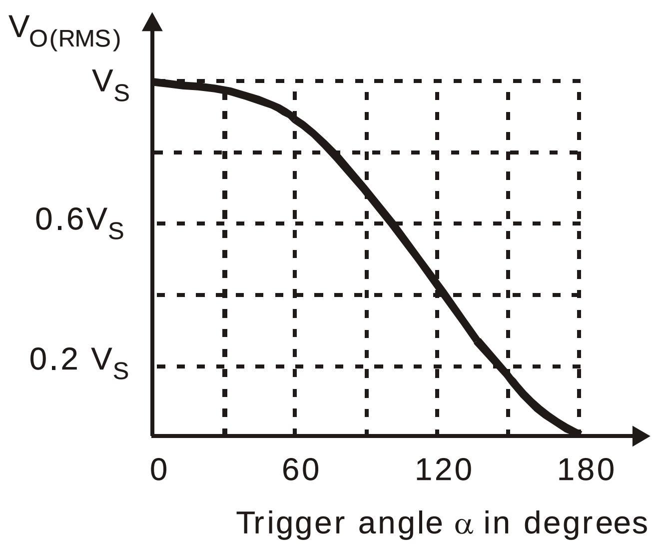
CONTROL CHARACTERISTIC OF SINGLE PHASE FULL-WAVE AC VOLTAGE CONTROLLER WITH RESISTIVE LOAD

The control characteristic is the plot of RMS output voltage $V_{O(RMS)}$ versus the trigger angle α; which can be obtained by using the expression for the RMS output voltage of a full-wave ac controller with resistive load.

$$V_{O(RMS)} = V_S\sqrt{\frac{1}{\pi}\left[(\pi - \alpha) + \frac{\sin 2\alpha}{2}\right]} \quad ;$$

Where $V_S = \frac{V_m}{\sqrt{2}}$ = RMS value of input supply voltage

Trigger angle α in degrees	**Trigger angle α in radians**	$V_{O(RMS)}$	**%**
0	0	V_S	100% V_S
30^0	$\pi/6$; $(1\pi/6)$	0.985477 V_S	98.54% V_S
60^0	$\pi/3$; $(2\pi/6)$	0.896938 V_S	89.69% V_S
90^0	$\pi/2$; $(3\pi/6)$	0.7071 V_S	70.7% V_S
120^0	$2\pi/3$; $(4\pi/6)$	0.44215 V_S	44.21% V_S
150^0	$5\pi/6$; $(5\pi/6)$	0.1698 V_S	16.98% V_S
180^0	π ; $(6\pi/6)$	0 V_S	0 V_S



We can notice from the figure, that we obtain a much better output control characteristic by using a single phase full wave ac voltage controller. The RMS output voltage can be varied from a maximum of 100% V_S at $\alpha = 0$ to a minimum of '0' at $\alpha = 180^0$. Thus we get a full range output voltage control by using a single phase full wave ac voltage controller.

Need For Isolation

In the single phase full wave ac voltage controller circuit using two SCRs or Thyristors T_1 and T_2 in parallel, the gating circuits (gate trigger pulse generating circuits) of Thyristors T_1 and T_2 must be isolated. Figure shows a pulse transformer with two separate windings to provide isolation between the gating signals of T_1 and T_2.

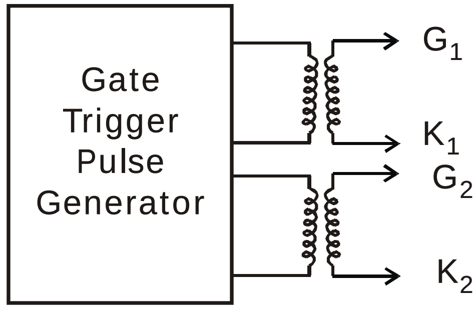


Fig.: Pulse Transformer

SINGLE PHASE FULL-WAVE AC VOLTAGE CONTROLLER WITH COMMON CATHODE

It is possible to design a single phase full wave ac controller with a common cathode configuration by having a common cathode point for T_1 and T_2 & by adding two diodes in a full wave ac controller circuit as shown in the figure below

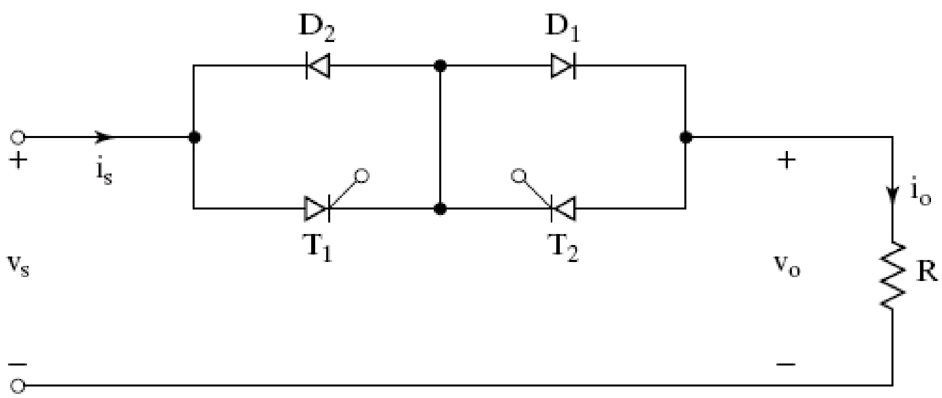


Fig.: Single phase full wave ac controller with common cathode (Bidirectional controller in common cathode configuration)

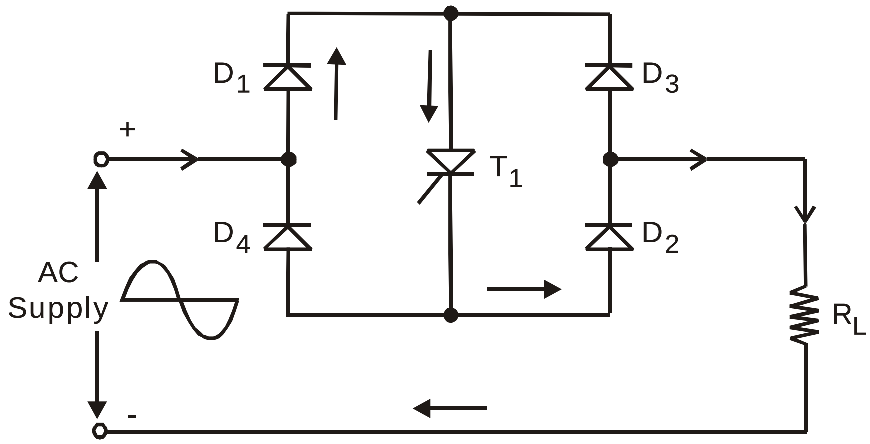
Thyristor T_1 and diode D_1 are forward biased during the positive half cycle of input supply. When thyristor T_1 is triggered at a delay angle α, Thyristor T_1 and diode D_1 conduct together from $\omega t = \alpha$ to π during the positive half cycle.

The thyristor T_2 and diode D_2 are forward biased during the negative half cycle of input supply, when trigged at a delay angle α, thyristor T_2 and diode D_2 conduct together during the negative half cycle from $\omega t = (\pi + \alpha)$ to 2π.

In this circuit as there is one single common cathode point, routing of the gate trigger pulses to the thyristor gates of T_1 and T_2 is simpler and only one isolation circuit is required.

But due to the need of two power diodes the costs of the devices increase. As there are two power devices conducting at the same time the voltage drop across the ON devices increases and the ON state conducting losses of devices increase and hence the efficiency decreases.

SINGLE PHASE FULL WAVE AC VOLTAGE CONTROLLER USING A SINGLE THYRISTOR



A single phase full wave ac controller can also be implemented with one thyristor and four diodes connected in a full wave bridge configuration as shown in the above figure. The four diodes act as a bridge full wave rectifier. The voltage across the thyristor T_1 and current through thyristor T_1 are always unidirectional. When T_1 is triggered at $\omega t = \alpha$, during the positive half cycle $\left(0 \le \alpha \le \pi\right)$, the load current flows through D_1, T_1, diode D_2 and through the load. With a resistive load, the thyristor current (flowing through the ON thyristor T_1) , the load current falls to zero at $\omega t = \pi$, when the input supply voltage decreases to zero at $\omega t = \pi$, the thyristor naturally turns OFF.

In the negative half cycle, diodes $D_3 \& D_4$ are forward biased during $\omega t = \pi$ to 2π radians. When T_1 is triggered at $\omega t = \left(\pi + \alpha\right)$, the load current flows in the opposite direction (upward direction) through the load, through D_3, T_1 and D_4. Thus D_3, D_4 and T_1 conduct together during the negative half cycle to supply the load power. When the input supply voltage becomes zero at $\omega t = 2\pi$, the thyristor current (load current) falls to zero at $\omega t = 2\pi$ and the thyristor T_1 naturally turns OFF. The waveforms and the expression for the RMS output voltage are the same as discussed earlier for the single phase full wave ac controller.

But however if there is a large inductance in the load circuit, thyristor T_1 may not be turned OFF at the zero crossing points, in every half cycle of input voltage and this may result in a loss of output control. This would require detection of the zero crossing of the load current waveform in order to ensure guaranteed turn off of the conducting thyristor before triggering the thyristor in the next half cycle, so that we gain control on the output voltage.

In this full wave ac controller circuit using a single thyristor, as there are three power devices conducting together at the same time there is more conduction voltage drop and an increase in the ON state conduction losses and hence efficiency is also reduced.

The diode bridge rectifier and thyristor (or a power transistor) act together as a bidirectional switch which is commercially available as a single device module and it has relatively low ON state conduction loss. It can be used for bidirectional load current control and for controlling the RMS output voltage.

SINGLE PHASE FULL WAVE AC VOLTAGE CONTROLLER (BIDIRECTIONAL CONTROLLER) WITH RL LOAD

In this section we will discuss the operation and performance of a single phase full wave ac voltage controller with RL load. In practice most of the loads are of RL type. For example if we consider a single phase full wave ac voltage controller controlling the speed of a single phase ac induction motor, the load which is the induction motor winding is an RL type of load, where R represents the motor winding resistance and L represents the motor winding inductance.

A single phase full wave ac voltage controller circuit (bidirectional controller) with an RL load using two thyristors T_1 and T_2 (T_1 and T_2 are two SCRs) connected in parallel is shown in the figure below. In place of two thyristors a single Triac can be used to implement a full wave ac controller, if a suitable Traic is available for the desired RMS load current and the RMS output voltage ratings.

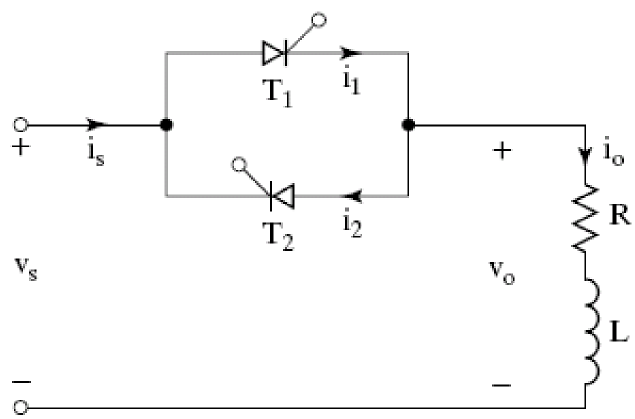


Fig: Single phase full wave ac voltage controller with RL load

The thyristor T_1 is forward biased during the positive half cycle of input supply. Let us assume that T_1 is triggered at $\omega t = \alpha$, by applying a suitable gate trigger pulse to T_1 during the positive half cycle of input supply. The output voltage across the load follows the input supply voltage when T_1 is ON. The load current i_O flows through the thyristor T_1 and through the load in the downward direction. This load current pulse flowing through T_1 can be considered as the positive current pulse. Due to the inductance in the load, the load current i_O flowing through T_1 would not fall to zero at $\omega t = \pi$, when the input supply voltage starts to become negative.

The thyristor T_1 will continue to conduct the load current until all the inductive energy stored in the load inductor L is completely utilized and the load current through T_1 falls to zero at $\omega t = \beta$, where β is referred to as the Extinction angle, (the value of ωt) at which the load current falls to zero. The extinction angle β is measured from the point of the beginning of the positive half cycle of input supply to the point where the load current falls to zero.

The thyristor T_1 conducts from $\omega t = \alpha$ to β. The conduction angle of T_1 is $\delta = (\beta - \alpha)$, which depends on the delay angle α and the load impedance angle ϕ. The waveforms of the input supply voltage, the gate trigger pulses of T_1 and T_2, the thyristor current, the load current and the load voltage waveforms appear as shown in the figure below.

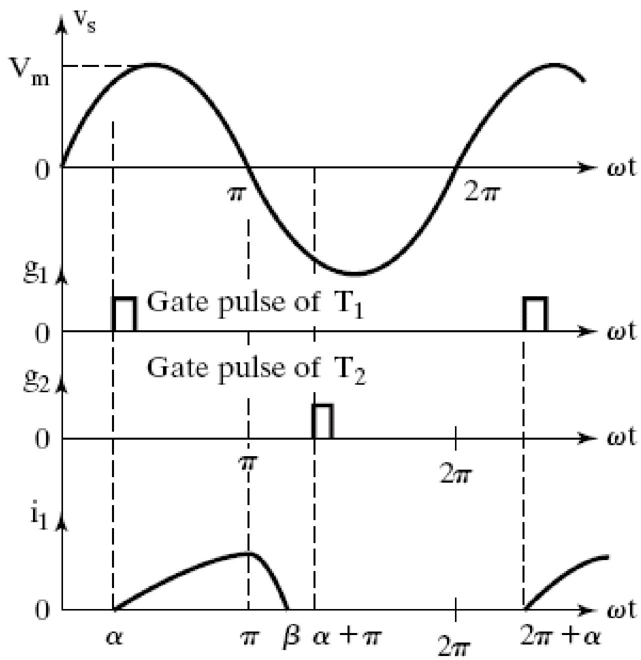


Fig.: Input supply voltage & Thyristor current waveforms

β is the extinction angle which depends upon the load inductance value.

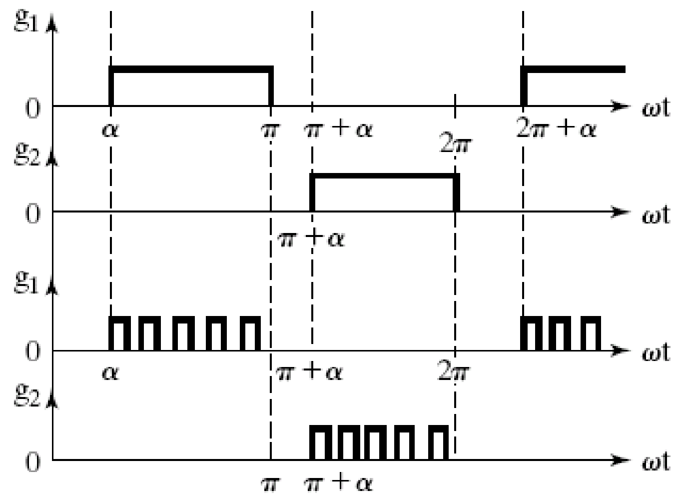
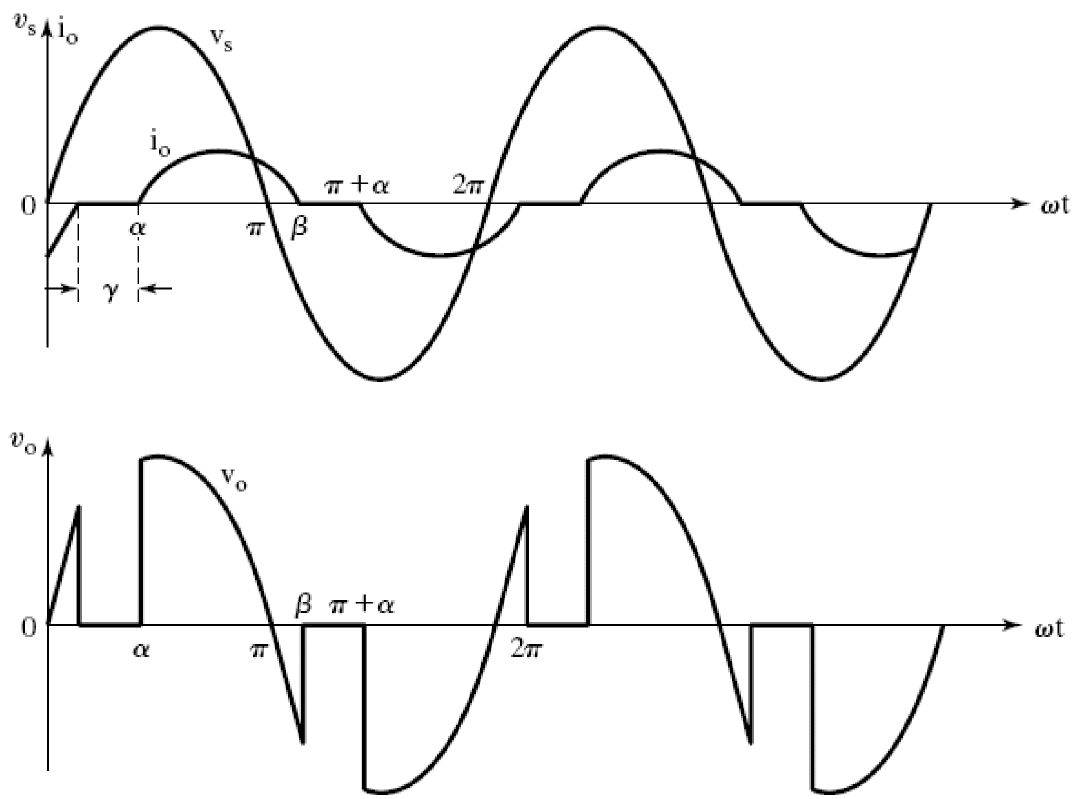


Fig.: Gating Signals

Waveforms of single phase full wave ac voltage controller with RL load for $\alpha > \phi$**. Discontinuous load current operation occurs for** $\alpha > \phi$ **and** $\beta < (\pi + \alpha)$**;**
i.e., $(\beta - \alpha) < \pi$**, conduction angle** $< \pi$**.**



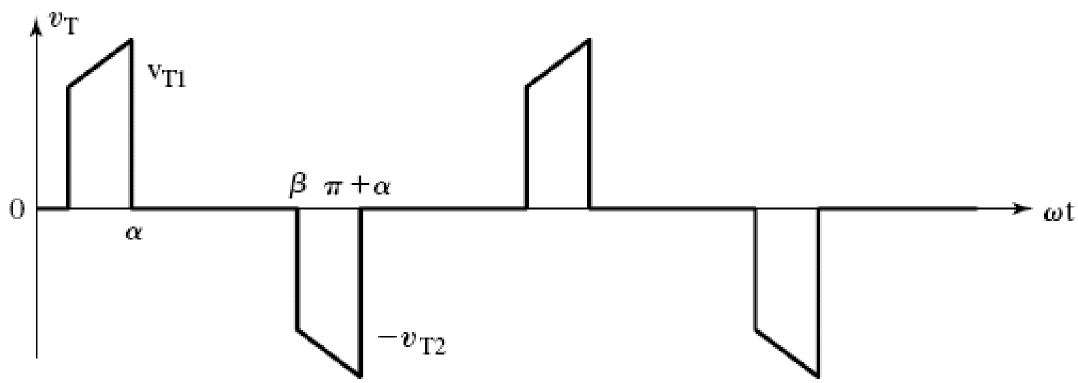


Fig.: Waveforms of Input supply voltage, Load Current, Load Voltage and Thyristor Voltage across T_1

Note

- The RMS value of the output voltage and the load current may be varied by varying the trigger angle α .
- This circuit, AC RMS voltage controller can be used to regulate the RMS voltage across the terminals of an ac motor (induction motor). It can be used to control the temperature of a furnace by varying the RMS output voltage.
- For very large load inductance 'L' the SCR may fail to commutate, after it is triggered and the load voltage will be a full sine wave (similar to the applied input supply voltage and the output control will be lost) as long as the gating signals are applied to the thyristors T_1 and T_2 . The load current waveform will appear as a full continuous sine wave and the load current waveform lags behind the output sine wave by the load power factor angle ϕ.

TO DERIVE AN EXPRESSION FOR THE OUTPUT (INDUCTIVE LOAD) CURRENT, DURING $\omega t = \alpha$ to β WHEN THYRISTOR T_1 CONDUCTS

Considering sinusoidal input supply voltage we can write the expression for the supply voltage as

$$v_S = V_m \sin \omega t \text{ = instantaneous value of the input supply voltage.}$$

Let us assume that the thyristor T_1 is triggered by applying the gating signal to T_1 at $\omega t = \alpha$. The load current which flows through the thyristor T_1 during $\omega t = \alpha$ to β can be found from the equation

$$L\left(\frac{di_O}{dt}\right) + Ri_O = V_m \sin \omega t \quad ;$$

The solution of the above differential equation gives the general expression for the output load current which is of the form

$$i_O = \frac{V_m}{Z}\sin(\omega t - \phi) + A_1 e^{\frac{-t}{\tau}} \quad ;$$

Where $V_m = \sqrt{2}V_S$ = maximum or peak value of input supply voltage.

$$Z = \sqrt{R^2 + (\omega L)^2} \text{ = Load impedance.}$$

$$\phi = \tan^{-1}\left(\frac{\omega L}{R}\right) \text{ = Load impedance angle (power factor angle of load).}$$

$$\tau = \frac{L}{R} \text{ = Load circuit time constant.}$$

Therefore the general expression for the output load current is given by the equation

$$i_O = \frac{V_m}{Z}\sin(\omega t - \phi) + A_1 e^{\frac{-R}{L}t} \quad ;$$

The value of the constant A_1 can be determined from the initial condition. i.e. initial value of load current $i_O = 0$, at $\omega t = \alpha$. Hence from the equation for i_O equating i_O to zero and substituting $\omega t = \alpha$, we get

$$i_O = 0 = \frac{V_m}{Z}\sin(\alpha - \phi) + A_1 e^{\frac{-R}{L}t}$$

Therefore $$A_1 e^{\frac{-R}{L}t} = \frac{-V_m}{Z}\sin(\alpha - \phi)$$

$$A_1 = \frac{1}{e^{\frac{-R}{L}t}}\left[\frac{-V_m}{Z}\sin(\alpha - \phi)\right]$$

$$A_1 = e^{\frac{+R}{L}t}\left[\frac{-V_m}{Z}\sin(\alpha - \phi)\right]$$

$$A_1 = e^{\frac{R(\omega t)}{\omega L}}\left[\frac{-V_m}{Z}\sin(\alpha - \phi)\right]$$

By substituting $\omega t = \alpha$, we get the value of constant A_1 as

$$A_1 = e^{\frac{R(\alpha)}{\omega L}}\left[\frac{-V_m}{Z}\sin(\alpha - \phi)\right]$$

Substituting the value of constant A_1 from the above equation into the expression for i_O, we obtain

$$i_O = \frac{V_m}{Z}\sin(\omega t-\phi) + e^{\frac{-R}{L}t} e^{\frac{R(\alpha)}{\omega L}}\left[\frac{-V_m}{Z}\sin(\alpha-\phi)\right] \;;$$

$$i_O = \frac{V_m}{Z}\sin(\omega t-\phi) + e^{\frac{-R(\omega t)}{\omega L}} e^{\frac{R(\alpha)}{\omega L}}\left[\frac{-V_m}{Z}\sin(\alpha-\phi)\right]$$

$$i_O = \frac{V_m}{Z}\sin(\omega t-\phi) + e^{\frac{-R}{\omega L}(\omega t-\alpha)}\left[\frac{-V_m}{Z}\sin(\alpha-\phi)\right]$$

Therefore we obtain the final expression for the inductive load current of a single phase full wave ac voltage controller with RL load as

$$i_O = \frac{V_m}{Z}\left[\sin(\omega t-\phi) - \sin(\alpha-\phi)e^{\frac{-R}{\omega L}(\omega t-\alpha)}\right] \;; \quad \text{Where } \alpha \le \omega t \le \beta .$$

The above expression also represents the thyristor current i_{T1}, during the conduction time interval of thyristor T_1 from $\omega t = \alpha$ to β.

To Calculate Extinction Angle β

The extinction angle β, which is the value of ωt at which the load current i_O falls to zero and T_1 is turned off can be estimated by using the condition that $i_O = 0$, at $\omega t = \beta$

By using the above expression for the output load current, we can write

$$i_O = 0 = \frac{V_m}{Z}\left[\sin(\beta-\phi) - \sin(\alpha-\phi)e^{\frac{-R}{\omega L}(\beta-\alpha)}\right]$$

As $\frac{V_m}{Z} \ne 0$ we can write

$$\left[\sin(\beta-\phi) - \sin(\alpha-\phi)e^{\frac{-R}{\omega L}(\beta-\alpha)}\right] = 0$$

Therefore we obtain the expression

$$\sin(\beta-\phi) = \sin(\alpha-\phi)e^{\frac{-R}{\omega L}(\beta-\alpha)}$$

The extinction angle β can be determined from this transcendental equation by using the iterative method of solution (trial and error method). After β is calculated, we can determine the thyristor conduction angle $\delta = (\beta - \alpha)$.

β is the extinction angle which depends upon the load inductance value. Conduction angle δ increases as α is decreased for a known value of β.

For $\delta < \pi$ radians, i.e., for $(\beta - \alpha) < \pi$ radians, for $(\pi + \alpha) > \beta$ the load current waveform appears as a discontinuous current waveform as shown in the figure. The output load current remains at zero during $\omega t = \beta$ to $(\pi + \alpha)$. This is referred to as discontinuous load current operation which occurs for $\beta < (\pi + \alpha)$.

When the trigger angle α is decreased and made equal to the load impedance angle ϕ i.e., when $\alpha = \phi$ we obtain from the expression for $\sin(\beta - \phi)$,

$$\sin(\beta - \phi) = 0 \;\; ; \;\; \text{Therefore } (\beta - \phi) = \pi \text{ radians.}$$

Extinction angle $\quad \beta = (\pi + \phi) = (\pi + \alpha) \;\; ; \;\;$ for the case when $\alpha = \phi$

Conduction angle $\quad \delta = (\beta - \alpha) = \pi \text{ radians} = 180^0 \;\; ; \;\;$ for the case when $\alpha = \phi$

Each thyristor conducts for 180^0 (π radians) . T_1 conducts from $\omega t = \phi$ to $(\pi + \phi)$ and provides a positive load current. T_2 conducts from $(\pi + \phi)$ to $(2\pi + \phi)$ and provides a negative load current. Hence we obtain a continuous load current and the output voltage waveform appears as a continuous sine wave identical to the input supply voltage waveform for trigger angle $\alpha \le \phi$ and the control on the output is lost.

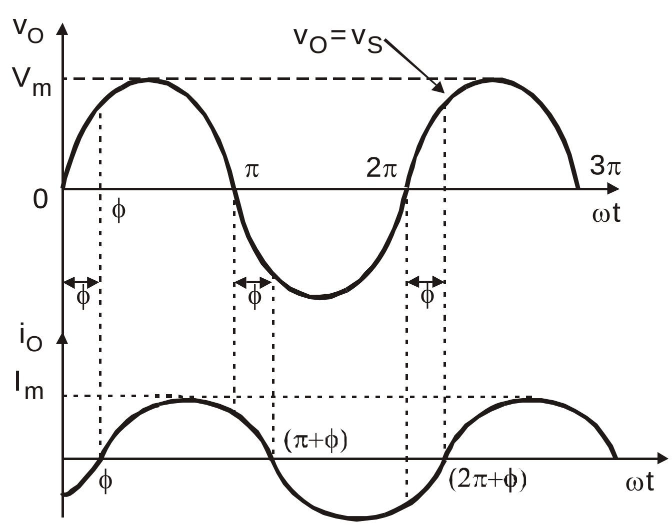


Fig.: Output voltage and output current waveforms for a single phase full wave ac voltage controller with RL load for $\alpha \le \phi$

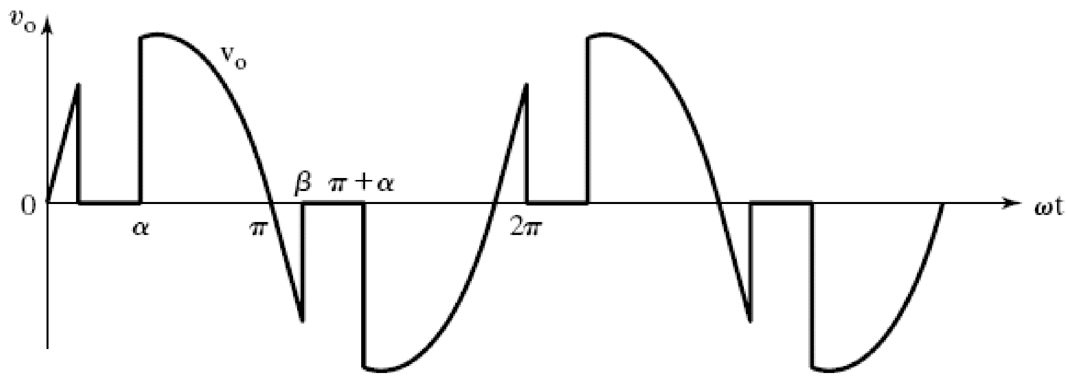
Thus we observe that for trigger angle $\alpha \le \phi$, the load current tends to flow continuously and we have continuous load current operation, without any break in the load current waveform and we obtain output voltage waveform which is a continuous sinusoidal waveform identical to the input supply voltage waveform. We loose the control on the output voltage for $\alpha \le \phi$ as the output voltage becomes equal to the input supply voltage and thus we obtain

$$V_{O(RMS)} = \frac{V_m}{\sqrt{2}} = V_S \quad ; \text{ for } \alpha \le \phi$$

Hence,

RMS output voltage = RMS input supply voltage for $\alpha \le \phi$

TO DERIVE AN EXPRESSION FOR RMS OUTPUT VOLTAGE $V_{O(RMS)}$ OF A SINGLE PHASE FULL-WAVE AC VOLTAGE CONTROLLER WITH RL LOAD.



When $\alpha > \varnothing$, the load current and load voltage waveforms become discontinuous as shown in the figure above.

$$V_{O(RMS)} = \left[\frac{1}{\pi}\int_{\alpha}^{\beta} V_m^{\ 2}\sin^2\omega t.d\left(\omega t\right)\right]^{\frac{1}{2}}$$

Output $v_o = V_m \sin\omega t$, for $\omega t = \alpha$ to β, when T_1 is ON.

$$V_{O(RMS)} = \left[\frac{V_m^{\ 2}}{\pi}\int_{\alpha}^{\beta}\frac{\left(1-\cos 2\omega t\right)}{2}d\left(\omega t\right)\right]^{1/2}$$

$$V_{O(RMS)} = \left[\frac{V_m^{\ 2}}{2\pi}\left\{\int_{\alpha}^{\beta} d\left(\omega t\right) - \int_{\alpha}^{\beta}\cos 2\omega t.d\left(\omega t\right)\right\}\right]^{1/2}$$

$$V_{O(RMS)} = \left[\frac{V_m^{\ 2}}{2\pi}\left\{\left(\omega t\right)\Big/_{\alpha}^{\beta} - \left(\frac{\sin 2\omega t}{2}\right)\Big/_{\alpha}^{\beta}\right\}\right]^{1/2}$$

$$V_{O(RMS)} = \left[\frac{V_m^{\ 2}}{2\pi}\left\{\left(\beta-\alpha\right) - \frac{\sin 2\beta}{2} + \frac{\sin 2\alpha}{2}\right\}\right]^{1/2}$$

$$V_{O(RMS)} = V_m\left[\frac{1}{2\pi}\left\{\left(\beta-\alpha\right) + \frac{\sin 2\alpha}{2} - \frac{\sin 2\beta}{2}\right\}\right]^{1/2}$$

$$V_{O(RMS)} = \frac{V_m}{\sqrt{2}}\left[\frac{1}{\pi}\left\{(\beta-\alpha)+\frac{\sin 2\alpha}{2}-\frac{\sin 2\beta}{2}\right\}\right]^{1/2}$$

The RMS output voltage across the load can be varied by changing the trigger angle α.

For a purely resistive load $L=0$, therefore load power factor angle $\phi=0$.

$$\phi = \tan^{-1}\left(\frac{\omega L}{R}\right)=0 \quad ;$$

Extinction angle $\quad \beta = \pi$ radians $=180^0$

PERFORMANCE PARAMETERS OF A SINGLE PHASE FULL WAVE AC VOLTAGE CONTROLLER WITH RESISTIVE LOAD

- **RMS Output Voltage** $V_{O(RMS)} = \frac{V_m}{\sqrt{2}}\sqrt{\frac{1}{\pi}\left[(\pi-\alpha)+\frac{\sin 2\alpha}{2}\right]}$; $\frac{V_m}{\sqrt{2}} = V_S$ = RMS input supply voltage.

- $I_{O(RMS)} = \frac{V_{O(RMS)}}{R_L}$ = RMS value of load current.

- $I_S = I_{O(RMS)}$ = RMS value of input supply current.

- **Output load power**

$$P_O = I^2_{O(RMS)} \times R_L$$

- **Input Power Factor**

$$PF = \frac{P_O}{V_S \times I_S} = \frac{I^2_{O(RMS)} \times R_L}{V_S \times I_{O(RMS)}} = \frac{I_{O(RMS)} \times R_L}{V_S}$$

$$PF = \frac{V_{O(RMS)}}{V_S} = \sqrt{\frac{1}{\pi}\left[(\pi-\alpha)+\frac{\sin 2\alpha}{2}\right]}$$

- **Average Thyristor Current,**

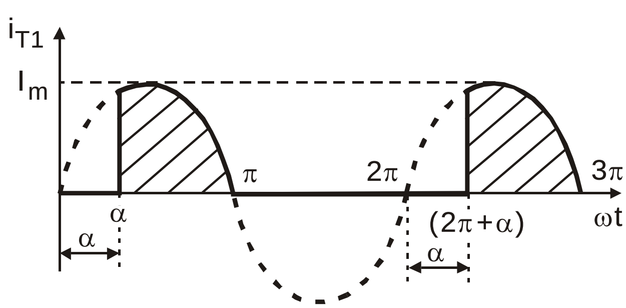


Fig.: Thyristor Current Waveform

$$I_{T(Avg)} = \frac{1}{2\pi}\int_{\alpha}^{\pi} i_T d(\omega t) = \frac{1}{2\pi}\int_{\alpha}^{\pi} I_m \sin\omega t.d(\omega t)$$

$$I_{T(Avg)} = \frac{I_m}{2\pi}\int_{\alpha}^{\pi}\sin\omega t.d(\omega t) = \frac{I_m}{2\pi}\left[-\cos\omega t\Big/_{\alpha}^{\pi}\right]$$

$$I_{T(Avg)} = \frac{I_m}{2\pi}\left[-\cos\pi + \cos\alpha\right] = \frac{I_m}{2\pi}\left[1+\cos\alpha\right]$$

- **Maximum Average Thyristor Current, for** $\alpha = 0$,

$$I_{T(Avg)} = \frac{I_m}{\pi}$$

- **RMS Thyristor Current**

$$I_{T(RMS)} = \sqrt{\frac{1}{2\pi}\left[\int_{\alpha}^{\pi} I_m^2 \sin^2\omega t.d(\omega t)\right]}$$

$$I_{T(RMS)} = \frac{I_m}{\sqrt{2}}\sqrt{\frac{1}{2\pi}\left[(\pi-\alpha)+\frac{\sin 2\alpha}{2}\right]}$$

- **Maximum RMS Thyristor Current, for** $\alpha = 0$,

$$I_{T(RMS)} = \frac{I_m}{2}$$

In the case of a single phase full wave ac voltage controller circuit using a Triac with resistive load, the average thyristor current $I_{T(Avg)} = 0$. Because the Triac conducts in both the half cycles and the thyristor current is alternating and we obtain a symmetrical thyristor current waveform which gives an average value of zero on integration.

PERFORMANCE PARAMETERS OF A SINGLE PHASE FULL WAVE AC VOLTAGE CONTROLLER WITH R-L LOAD

The Expression for the Output (Load) Current

The expression for the output (load) current which flows through the thyristor, during $\omega t = \alpha$ to β is given by

$$i_O = i_{T_1} = \frac{V_m}{Z}\left[\sin(\omega t-\phi) - \sin(\alpha-\phi)e^{\frac{-R}{\omega L}(\omega t-\alpha)}\right] \quad ; \quad \text{for } \alpha \le \omega t \le \beta$$

Where,

$V_m = \sqrt{2}V_S$ = Maximum or peak value of input ac supply voltage.

$Z = \sqrt{R^2 + (\omega L)^2}$ = Load impedance.

$\phi = \tan^{-1}\left(\frac{\omega L}{R}\right)$ = Load impedance angle (load power factor angle).

α = Thyristor trigger angle = Delay angle.

β = Extinction angle of thyristor, (value of ωt) at which the thyristor (load) current falls to zero.

β is calculated by solving the equation

$$\sin\left(\beta - \phi\right) = \sin\left(\alpha - \phi\right) e^{\frac{-R}{\omega L}(\beta - \alpha)}$$

Thyristor Conduction Angle $\delta = \left(\beta - \alpha\right)$

Maximum thyristor conduction angle $\delta = \left(\beta - \alpha\right) = \pi$ radians = 180^0 for $\alpha \le \phi$.

RMS Output Voltage

$$V_{O(RMS)} = \frac{V_m}{\sqrt{2}} \sqrt{\frac{1}{\pi}\left[\left(\beta - \alpha\right) + \frac{\sin 2\alpha}{2} - \frac{\sin 2\beta}{2}\right]}$$

The Average Thyristor Current

$$I_{T(Avg)} = \frac{1}{2\pi}\left[\int_{\alpha}^{\beta} i_{T_1} d\left(\omega t\right)\right]$$

$$I_{T(Avg)} = \frac{1}{2\pi}\left[\int_{\alpha}^{\beta} \frac{V_m}{Z}\left[\sin\left(\omega t - \phi\right) - \sin\left(\alpha - \phi\right) e^{\frac{-R}{\omega L}(\omega t - \alpha)}\right] d\left(\omega t\right)\right]$$

$$I_{T(Avg)} = \frac{V_m}{2\pi Z}\left[\int_{\alpha}^{\beta} \sin\left(\omega t - \phi\right).d\left(\omega t\right) - \int_{\alpha}^{\beta} \sin\left(\alpha - \phi\right) e^{\frac{-R}{\omega L}(\omega t - \alpha)} d\left(\omega t\right)\right]$$

Maximum value of $I_{T(Avg)}$ occur at $\alpha = 0$. The thyristors should be rated for maximum $I_{T(Avg)} = \left(\frac{I_m}{\pi}\right)$, where $I_m = \frac{V_m}{Z}$.

RMS Thyristor Current $I_{T(RMS)}$

$$I_{T(RMS)} = \sqrt{\left[\frac{1}{2\pi}\int_{\alpha}^{\beta} i_{T_1}^2 d\left(\omega t\right)\right]}$$

Maximum value of $I_{T(RMS)}$ occurs at $\alpha = 0$. Thyristors should be rated for maximum $I_{T(RMS)} = \left(\frac{I_m}{2}\right)$

When a Triac is used in a single phase full wave ac voltage controller with RL type of load, then $I_{T(Avg)} = 0$ and maximum $I_{T(RMS)} = \frac{I_m}{\sqrt{2}}$

PROBLEMS

1. *A single phase full wave ac voltage controller supplies an RL load. The input supply voltage is 230V, RMS at 50Hz. The load has L = 10mH, R = 10Ω, the delay angle of thyristors T_1 and T_2 are equal, where $\alpha_1 = \alpha_2 = \frac{\pi}{3}$. Determine*
    a. *Conduction angle of the thyristor T_1.*
    b. *RMS output voltage.*
    c. *The input power factor.*
    Comment on the type of operation.

2. *A single phase full wave controller has an input voltage of 120 V (RMS) and a load resistance of 6 ohm. The firing angle of thyristor is $\pi/2$. Find*
    a. *RMS output voltage*
    b. *Power output*
    c. *Input power factor*
    d. *Average and RMS thyristor current.*

3. *A single phase half wave ac regulator using one SCR in anti-parallel with a diode feeds 1 kW, 230 V heater. Find load power for a firing angle of 45^0.*

4. *Find the RMS and average current flowing through the heater shown in figure. The delay angle of both the SCRs is 45^0.*