DC – DC CONVERTER
(DC - CHOPPER

ASNIL
ELEKTRO FT UNP
BLOCK DIAGRAM OF DC-DC CONVERTERS

- AC line voltage (1-phase or 3-phase)
- Uncontrolled Diode Rectifier
- DC (unregulated)
- Filter Capacitor
- DC (unregulated)
- DC-DC Converter
- DC (regulated)
- Load
- Battery
- $V_{\text{control}}$
A chopper is a static device which is used to obtain a variable dc voltage from a constant dc voltage source. A chopper is also known as dc-to-dc converter. The thyristor converter offers greater efficiency, faster response, lower maintenance, smaller size and smooth control. Choppers are widely used in trolley cars, battery operated vehicles, traction motor control, control of large number of dc motors, etc..... They are also used in regenerative braking of dc motors to return energy back to supply and also as dc voltage regulators.
Choppers are of two types:
  Step-down choppers
  Step-up choppers

In step-down choppers, the output voltage will be less than the input voltage whereas in step-up choppers output voltage will be more than the input voltage.

**PRINCIPLE OF STEP-DOWN CHOPPER**

![Diagram of step-down chopper with resistive load](image)

*Fig. 2.1: Step-down Chopper with Resistive Load*
Figure 2.1 shows a step-down chopper with resistive load. The thyristor in the circuit acts as a switch. When thyristor is ON, supply voltage appears across the load and when thyristor is OFF, the voltage across the load will be zero. The output voltage and current waveforms are as shown in figure 2.2.

![Diagram of step-down chopper waveforms](image)

Fig. 2.2: Step-down choppers — output voltage and current waveforms
\[ V_{dc} = \text{average value of output or load voltage} \]

\[ I_{dc} = \text{average value of output or load current} \]

\[ t_{ON} = \text{time interval for which SCR conducts} \]

\[ t_{OFF} = \text{time interval for which SCR is OFF.} \]

\[ T = t_{ON} + t_{OFF} = \text{period of switching or chopping period} \]

\[ f = \frac{1}{T} = \text{frequency of chopper switching or chopping frequency.} \]

**Average output voltage**

\[ V_{dc} = V \left( \frac{t_{ON}}{t_{ON} + t_{OFF}} \right) \quad \cdots (2.1) \]

\[ V_{dc} = V \left( \frac{t_{ON}}{T} \right) = V \cdot d \quad \cdots (2.2) \]

but \[ \left( \frac{t_{ON}}{t} \right) = d = \text{duty cycle} \quad \cdots (2.3) \]
Average output current,

\[ I_{dc} = \frac{V_{dc}}{R} \tag{2.4} \]

\[ I_{dc} = \frac{V}{R} \left( \frac{t_{ON}}{T} \right) = \frac{V}{R} d \tag{2.5} \]

RMS value of output voltage

\[ V_o = \sqrt{\frac{1}{T} \int_{0}^{t_{ON}} v_o^2 dt} \]

But during \( t_{ON}, \ v_o = V \)

Therefore RMS output voltage

\[ V_o = \sqrt{\frac{1}{T} \int_{0}^{t_{ON}} V^2 dt} \]

\[ V_o = \sqrt{\frac{V^2}{T} t_{ON}} = \sqrt{\frac{t_{ON}}{T}} V \tag{2.6} \]

\[ V_o = \sqrt{d} V \tag{2.7} \]
Output power\[ P_o = V_o I_o \]

But\[ I_o = \frac{V_o}{R} \]

Therefore output power\[ P_o = \frac{V_o^2}{R} \]
\[ P_o = \frac{dV^2}{R} \quad \text{...(2.8)} \]

Effective input resistance of chopper\[ R_i = \frac{V}{I_{dc}} \quad \text{...(2.9)} \]
\[ R_i = \frac{R}{d} \quad \text{...(2.10)} \]

The output voltage can be varied by varying the duty cycle.
METHODS OF CONTROL

The output dc voltage can be varied by the following methods.

- Pulse width modulation control or constant frequency operation.
- Variable frequency control.

PULSE WIDTH MODULATION

In pulse width modulation the pulse width \((t_{ON})\) of the output waveform is varied keeping chopping frequency ‘\(f\)’ and hence chopping period ‘\(T\)’ constant. Therefore output voltage is varied by varying the ON time, \(t_{ON}\). Figure 2.3 shows the output voltage waveforms for different ON times.

VARIABLE FREQUENCY CONTROL

In this method of control, chopping frequency \(f\) is varied keeping either \(t_{ON}\) or \(t_{OFF}\) constant. This method is also known as frequency modulation.

Figure 2.4 shows the output voltage waveforms for a constant \(t_{ON}\) and variable chopping period \(T\).

In frequency modulation to obtain full output voltage, range frequency has to be varied over a wide range. This method produces harmonics in the output and for large \(t_{OFF}\) load current may become discontinuous.
Fig. 2.3: Pulse Width Modulation Control
Fig. 2.4: Output Voltage Waveforms for Time Ratio Control
STEP-DOWN CHOPPER WITH R-L LOAD

Figure 2.5 shows a step-down chopper with R-L load and free wheeling diode. When chopper is ON, the supply is connected across the load. Current flows from the supply to the load. When chopper is OFF, the load current $i_o$ continues to flow in the same direction through the free-wheeling diode due to the energy stored in the inductor L. The load current can be continuous or discontinuous depending on the values of L and duty cycle, d. For a continuous current operation the load current is assumed to vary between two limits $I_{\text{min}}$ and $I_{\text{max}}$.

Figure 2.6 shows the output current and output voltage waveforms for a continuous current and discontinuous current operation.
Fig. 2.5: Step Down Chopper with R-L Load
When the current exceeds $I_{\text{max}}$ the chopper is turned-off and it is turned-on when current reduces to $I_{\text{min}}$.
EXPRESSIONS FOR LOAD CURRENT $i_o$ FOR CONTINUOUS CURRENT OPERATION WHEN CHOPPER IS ON ($0 \leq t \leq t_{ON}$)

![Circuit Diagram]

Fig. 2.5 (a)

Voltage equation for the circuit shown in figure 2.5(a) is

$$V = i_o R + L \frac{di_o}{dt} + E \quad \ldots (2.11)$$

Taking Laplace Transform

$$\frac{V}{S} = RI_o(S) + L \left[ S I_o(S) - i_o(0^-) \right] + \frac{E}{S} \quad \ldots (2.12)$$
At \( t = 0 \), initial current \( i_O(0^-) = I_{\text{min}} \)

\[
I_O(S) = \frac{V - E}{LS\left(S + \frac{R}{L}\right)} + \frac{I_{\text{min}}}{S + \frac{R}{L}} \quad \ldots (2.13)
\]

Taking Inverse Laplace Transform

\[
i_O(t) = \frac{V - E}{R} \left[1 - e^{-\left(\frac{R}{L}\right)t}\right] + I_{\text{min}} e^{-\left(\frac{R}{L}\right)t} \quad \ldots (2.14)
\]

This expression is valid for \( 0 \leq t \leq t_{ON} \). i.e., during the period chopper is ON.

At the instant the chopper is turned off, load current is

\[
i_O(t_{ON}) = I_{\text{max}}
\]
When Chopper is OFF \((0 \leq t \leq t_{OFF})\)

\[ 0 = Ri_O + L \frac{di_O}{dt} + E \]

Voltage equation for the circuit shown in figure 2.5(b) is

\[ 0 = Ri_O (S) + L \left[ SI_O (S) - i_O (0^-) \right] + \frac{E}{S} \]
Redefining time origin we have at \( t = 0 \), initial current \( i_o(0^-) = I_{\text{max}} \)

Therefore

\[
I_o(S) = \frac{I_{\text{max}}}{S + \frac{R}{L}} - \frac{E}{L S\left(S + \frac{R}{L}\right)}
\]

Taking Inverse Laplace Transform

\[
i_o(t) = I_{\text{max}} e^{-\frac{R}{L} t} - \frac{E}{R} \left[1 - e^{-\frac{R}{L} t}\right]
\] \( \ldots (2.16) \)

The expression is valid for \( 0 \leq t \leq t_{\text{OFF}} \), i.e., during the period chopper is OFF. At the instant the chopper is turned ON or at the end of the off period, the load current is

\[
i_o(t_{\text{OFF}}) = I_{\text{min}}
\]
TO FIND $I_{\text{max}}$ AND $I_{\text{min}}$

From equation (2.14),

At 

$$t = t_{\text{ON}} = d \frac{L}{R}, \quad i_o(t) = I_{\text{max}}$$

Therefore 

$$I_{\text{max}} = \frac{V - E}{R} \left[ 1 - e^{-\frac{dRT}{L}} \right] + I_{\text{min}} e^{-\frac{dRT}{L}} \quad \ldots (2.17)$$

From equation (2.16),

At 

$$t = t_{\text{OFF}} = T - t_{\text{ON}}, \quad i_o(t) = I_{\text{min}}$$

$$t = t_{\text{OFF}} = (1 - d) T$$

Therefore 

$$I_{\text{min}} = I_{\text{max}} e^{-\frac{(1-d)RT}{L}} - \frac{E}{R} \left[ 1 - e^{-\frac{(1-d)RT}{L}} \right] \quad \ldots (2.18)$$

Substituting for $I_{\text{min}}$ in equation (2.17) we get,

$$I_{\text{max}} = \frac{V}{R} \left[ 1 - e^{-\frac{dRT}{L}} \right] - \frac{E}{R} \left[ 1 - e^{-\frac{dRT}{L}} \right] \quad \ldots (2.19)$$
Substituting for $I_{\text{max}}$ in equation (2.18) we get,

$$I_{\text{min}} = \frac{V}{R} \left[ \frac{dRT}{e^{\frac{RT}{L}} - 1} - \frac{E}{R} \right]$$

\[ ...(2.20) \]

$(I_{\text{max}} - I_{\text{min}})$ is known as the steady state ripple.

Therefore peak-to-peak ripple current

$$\Delta I = I_{\text{max}} - I_{\text{min}}$$

Average output voltage

$$V_{dc} = d \cdot V$$

\[ ...(2.21) \]

Average output current

$$I_{dc(\text{approx})} = \frac{I_{\text{max}} + I_{\text{min}}}{2}$$

\[ ...(2.22) \]
Assuming load current varies linearly from $I_{\text{min}}$ to $I_{\text{max}}$ instantaneous load current is given by

$$i_o = I_{\text{min}} + \frac{(\Delta I) t}{dT} \text{ for } 0 \leq t \leq t_{ON}(dT)$$

$$i_o = I_{\text{min}} + \left(\frac{I_{\text{max}} - I_{\text{min}}}{dT}\right) t \quad \cdots (2.23)$$

RMS value of load current

$$I_{O(RMS)} = \sqrt{\frac{1}{dT} \int_{0}^{dT} i_o^2 dt}$$

$$I_{O(RMS)} = \sqrt{\frac{1}{dT} \int_{0}^{dT} \left[I_{\text{min}} + \left(\frac{I_{\text{max}} - I_{\text{min}}}{dT}\right) t\right]^2 dt}$$

$$I_{O(RMS)} = \sqrt{\frac{1}{dT} \int_{0}^{dT} \left[I_{\text{min}}^2 + \left(\frac{I_{\text{max}} - I_{\text{min}}}{dT}\right)^2 t^2 + \frac{2I_{\text{min}} (I_{\text{max}} - I_{\text{min}}) t}{dT}\right] dt}$$
RMS value of output current

\[ I_{O(RMS)} = \left[ I_{\min}^2 + \frac{(I_{\max} - I_{\min})^2}{3} + I_{\min} (I_{\max} - I_{\min}) \right]^{\frac{1}{2}} \] ... (2.24)

RMS chopper current

\[ I_{CH} = \sqrt{\frac{1}{T} \int_0^T i_0^2 dt} \]

\[ I_{CH} = \sqrt{\frac{1}{T} \int_0^T \left[ I_{\min} + \frac{(I_{\max} - I_{\min})}{dT} \right] dt} \]

\[ I_{CH} = \sqrt{d} \left[ I_{\min}^2 + \frac{(I_{\max} - I_{\min})^2}{3} + I_{\min} (I_{\max} - I_{\min}) \right]^{\frac{1}{2}} \]

\[ I_{CH} = \sqrt{d} I_{O(RMS)} \] ... (2.25)

Effective input resistance is

\[ R_i = \frac{V}{I_s} \]
Where $I_s =$ Average source current

$$I_s = dI_{dc}$$

Therefore

$$R_i = \frac{V}{dI_{dc}}$$  \hspace{1cm} \ldots(2.26)$$

**PRINCIPLE OF STEP-UP CHOPPER**

![Step-up Chopper Diagram](Image)

**Fig. 2.13:** Step-up Chopper
Figure 2.13 shows a step-up chopper to obtain a load voltage $V_0$ higher than the input voltage $V$. The values of $L$ and $C$ are chosen depending upon the requirement of output voltage and current. When the chopper is ON, the inductor $L$ is connected across the supply. The inductor current $I$ rises and the inductor stores energy during the ON time of the chopper, $t_{ON}$. When the chopper is off, the inductor current $I$ is forced to flow through the diode $D$ and load for a period, $t_{OFF}$. The current tends to decrease resulting in reversing the polarity of induced EMF in $L$. Therefore voltage across load is given by

$$V_0 = V + L \frac{dI}{dt} \quad \text{i.e.,} \quad V_0 > V \quad \text{...(2.27)}$$
EXPRESSIO FOR OUTPUT VOLTAGE

Assume the average inductor current to be \( I \) during ON and OFF time of Chopper.

When Chopper is ON

Voltage across inductor \( L = V \)

Therefore energy stored in inductor = \( V \cdot I \cdot t_{ON} \) \( \ldots (2.28) \),

where \( t_{ON} = ON \) period of chopper.

When Chopper is OFF (energy is supplied by inductor to load)

Voltage across \( L = V_o - V \)

Energy supplied by inductor \( L = (V_o - V) \cdot t_{OFF} \), where \( t_{OFF} = OFF \) period of Chopper.

Neglecting losses, energy stored in inductor \( L = \) energy supplied by inductor \( L \)

Therefore \( V \cdot I \cdot t_{ON} = (V_o - V) \cdot t_{OFF} \)

\( V_o = \frac{V \left( t_{ON} + t_{OFF} \right)}{t_{OFF}} \)
\[ V_o = V \left( \frac{T}{T - t_{ON}} \right) \]

Where

\[ T = \text{Chopping period or period of switching}. \]
\[ T = t_{ON} + t_{OFF} \]

\[ V_o = V \left( \frac{1}{1 - \frac{t_{ON}}{T}} \right) \]

Therefore

\[ V_o = V \left( \frac{1}{1 - d} \right) \quad \text{...(2.29)} \]

Where

\[ d = \frac{t_{ON}}{T} = \text{duty cycle} \]

For variation of duty cycle ‘d’ in the range of \( 0 < d < 1 \) the output voltage \( V_o \) will vary in the range \( V < V_o < \infty \).
CLASSIFICATION OF CHOPPERS

Choppers are classified as follows
- Class A Chopper
- Class B Chopper
- Class C Chopper
- Class D Chopper
- Class E Chopper

CLASS A CHOPPER

Fig. 2.14: Class A Chopper and \(v_0 - i_0\) Characteristic
Figure 2.14 shows a *Class A Chopper* circuit with inductive load and free-wheeling diode. When chopper is ON, supply voltage $V$ is connected across the load i.e., $v_o = V$ and current $i_o$ flows as shown in figure. When chopper is OFF, $v_o = 0$ and the load current $i_o$ continues to flow in the same direction through the free wheeling diode. Therefore the average values of output voltage and current i.e., $v_o$ and $i_o$ are always positive. Hence, *Class A Chopper* is a first quadrant chopper (or single quadrant chopper). Figure 2.15 shows output voltage and current waveforms for a continuous load current.
Fig. 2.15: First quadrant Chopper - Output Voltage and Current Waveforms
Class A Chopper is a step-down chopper in which power always flows from source to load. It is used to control the speed of dc motor. The output current equations obtained in step down chopper with R-L load can be used to study the performance of Class A Chopper.

Fig. 2.16: Class B Chopper
Fig. 2.16 shows a *Class B Chopper* circuit. When chopper is ON, \( v_o = 0 \) and \( E \) drives a current \( i_o \) through \( L \) and \( R \) in a direction opposite to that shown in figure 2.16. During the ON period of the chopper, the inductance \( L \) stores energy. When Chopper is OFF, diode \( D \) conducts, \( v_o = V \) and part of the energy stored in inductor \( L \) is returned to the supply. Also the current \( i_o \) continues to flow from the load to source. Hence the average output voltage is positive and average output current is negative. Therefore *Class B Chopper* operates in second quadrant. In this chopper, power flows from load to source. *Class B Chopper* is used for regenerative braking of dc motor. Figure 2.17 shows the output voltage and current waveforms of a *Class B Chopper*.

The output current equations can be obtained as follows. During the interval diode ‘\( D \)’ conducts (chopper is off) voltage equation is given by
\[ V = \frac{Ldi_o}{dt} + Ri_o + E \]

For the initial condition i.e., \( i_o(t) = I_{\text{min}} \) at \( t = 0 \).

The solution of the above equation is obtained along similar lines as in step-down chopper with R-L load.

Therefore

\[ i_o(t) = \frac{V - E}{R} \left( 1 - e^{-\frac{R}{L}t} \right) + I_{\text{min}} e^{\frac{R}{L}t} \quad 0 < t < t_{\text{OFF}} \]

At \( t = t_{\text{OFF}} \)

\[ i_o(t) = I_{\text{max}} \]

\[ I_{\text{max}} = \frac{V - E}{R} \left( 1 - e^{-\frac{R}{L}t_{\text{OFF}}} \right) + I_{\text{min}} e^{-\frac{R}{L}t_{\text{OFF}}} \]
During the interval chopper is ON voltage equation is given by

\[ 0 = \frac{Ldzi_o}{dt} + Ri_o + E \]

Redefining the time origin, at \( t = 0 \) \( i_o(t) = I_{\text{max}} \).

The solution for the stated initial condition is

\[ i_o(t) = I_{\text{max}} e^{-\frac{R}{L}t} - \frac{E}{R} \left( 1 - e^{-\frac{R}{L}t} \right) \quad \text{for} \quad 0 < t < t_{\text{ON}} \]

At \( t = t_{\text{ON}} \), \( i_o(t) = I_{\text{min}} \)

Therefore

\[ I_{\text{min}} = I_{\text{max}} e^{-\frac{R}{L}t_{\text{ON}}} - \frac{E}{R} \left( 1 - e^{-\frac{R}{L}t_{\text{ON}}} \right) \]
Fig. 2.17: Class B Chopper - Output Voltage and Current Waveforms
CLASS C CHOPPER

*Class C Chopper* is a combination of *Class A* and *Class B Choppers*. Figure 2.18 shows a *Class C* two quadrant Chopper circuit. For first quadrant operation, $CH_1$ is ON or $D_2$ conducts and for second quadrant operation, $CH_2$ is ON or $D_1$ conducts. When $CH_1$ is ON, the load current $i_O$ is positive, i.e., $i_O$ flows in the direction as shown in figure 2.18.

The output voltage is equal to $V(v_o = V)$ and the load receives power from the source.

![Figure 2.18: Class C Chopper](image-url)
When $CH_1$ is turned OFF, energy stored in inductance $L$ forces current to flow through the diode $D_2$ and the output voltage $v_o = 0$, but $i_o$ continues to flow in positive direction. When $CH_2$ is triggered, the voltage $E$ forces $i_o$ to flow in opposite direction through $L$ and $CH_2$. The output voltage $v_o = 0$. On turning OFF $CH_2$, the energy stored in the inductance drives current through diode $D_1$ and the supply; output voltage $v_o = V$ the input current becomes negative and power flows from load to source.

Thus the average output voltage $v_o$ is positive but the average output current $i_o$ can take both positive and negative values. Choppers $CH_1$ and $CH_2$ should not be turned ON simultaneously as it would result in short circuiting the supply. *Class C Chopper* can be used both for dc motor control and regenerative braking of dc motor. Figure 2.19 shows the output voltage and current waveforms.
Fig. 2.19: Class C Chopper - Output Voltage and Current Waveforms
Figure 2.20: Class D Chopper

Figure 2.20 shows a class D two quadrant chopper circuit. When both $CH_1$ and $CH_2$ are triggered simultaneously, the output voltage $v_O = V$ and output current $i_O$ flows through the load in the direction shown in figure 2.20. When $CH_1$ and $CH_2$ are turned OFF, the load current $i_O$ continues to flow in the same direction through load, $D_1$ and $D_2$, due to the energy stored in the inductor L, but output voltage $v_O = -V$. The average load voltage $v_O$ is positive if chopper ON-time ($t_{ON}$) is more than their OFF-time ($t_{OFF}$) and average output voltage becomes negative if $t_{ON} < t_{OFF}$. Hence the direction of load current is always positive but load voltage can be positive or negative. Waveforms are shown in figures 2.21 and 2.22.
Fig. 2.21: Output Voltage and Current Waveforms for $t_{ON} > t_{OFF}$
Fig. 2.22: Output Voltage and Current Waveforms for $t_{ON} < t_{OFF}$
CLASS E CHOPPER

Fig. 2.23: Class E Chopper

CH₂ - D₄ Conducts
D₁ - D₄ Conducts

CH₃ - CH₂ ON
CH₁ - CH₄ ON
CH₄ - D₂ Conducts

Fig. 2.23(a): Four Quadrant Operation
Figure 2.23 shows a class E 4 quadrant chopper circuit. When $CH_1$ and $CH_4$ are triggered, output current $i_o$ flows in positive direction as shown in figure 2.23 through $CH_1$ and $CH_4$, with output voltage $v_o = V$. This gives the first quadrant operation. When both $CH_1$ and $CH_4$ are OFF, the energy stored in the inductor L drives $i_o$ through $D_3$ and $D_2$ in the same direction, but output voltage $v_o = -V$. Therefore the chopper operates in the fourth quadrant. For fourth quadrant operation the direction of battery must be reversed. When $CH_2$ and $CH_3$ are triggered, the load current $i_o$ flows in opposite direction and output voltage $v_o = -V$.

Since both $i_o$ and $v_o$ are negative, the chopper operates in third quadrant. When both $CH_2$ and $CH_3$ are OFF, the load current $i_o$ continues to flow in the same direction through $D_1$ and $D_4$ and the output voltage $v_o = V$. Therefore the chopper operates in second quadrant as $v_o$ is positive but $i_o$ is negative. Figure 2.23(a) shows the devices which are operative in different quadrants.
Exercise

A Chopper circuit is operating on TRC at a frequency of 2 kHz on a 460 V supply. If the load voltage is 350 volts, calculate the conduction period of the thyristor in each cycle.
Solution

\[ V = 460 \text{ V}, \quad V_{dc} = 350 \text{ V}, \quad f = 2 \text{ kHz} \]

Chopping period

\[ T = \frac{1}{f} \]

\[ T = \frac{1}{2 \times 10^{-3}} = 0.5 \text{ msec} \]

Output voltage

\[ V_{dc} = \left( \frac{t_{ON}}{T} \right) V \]

Conduction period of thyristor

\[ t_{ON} = \frac{T \times V_{dc}}{V} \]

\[ t_{ON} = \frac{0.5 \times 10^{-3} \times 350}{460} \]

\[ t_{ON} = 0.38 \text{ msec} \]